Network Analysis:

The Hidden Structures behind the Webs We Weave 17-338 / 17-668

Triads and Structural Balance

Tuesday, September 9, 2025

Patrick Park



Quick Recap — Last Tuesday's Lecture

Social ties reflect graph structure

Graph structure reflects nature of the social tie and its context

Reciprocity and index of mutuality

Social ties and information diffusion

Tie persistence and decay

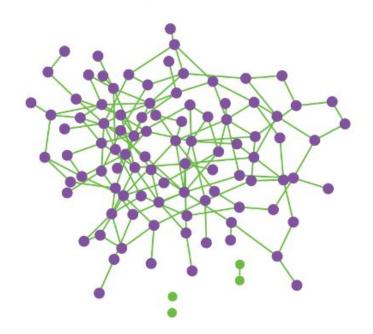
Larger Building Blocks: The Triad

Random graphs are useful as a baseline model.

But real-world social networks differ from random graphs in an important way.

They contain more triangles.

These triangles, or connected triads, are the telltale sign of social groups.



Probability of a triangle is close to 0 in random graphs.

How is a trio different from a duo?

Dyad is not a group:

- Not perceived as a social group
- Variable and capricious
- Individuality and emotion of the two dictate the relationship

Triad is a group:

- The smallest objectified social group ("me" and "them")
- The other two can pressure an individual to comply
- Can reduce caprice and uncertainty
- Individual complies out of fear of exclusion
- Individuality is suppressed and the group is cohesive

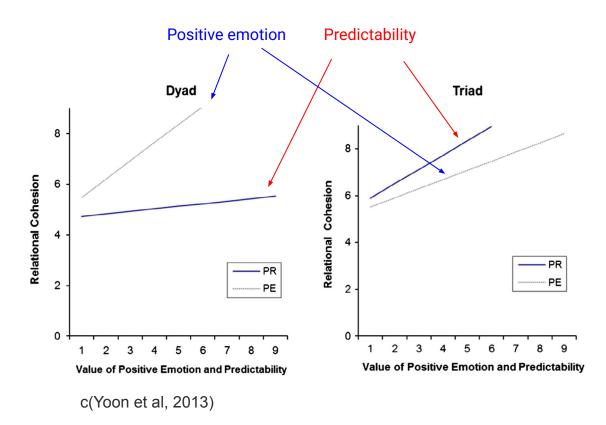


"each of the two [in a dyad] feels himself confronted only by the other, not by a collectivity above him" -Georg Simmel

Experimental subjects were divided into either a 2-person exchange or a 3-person exchange condition

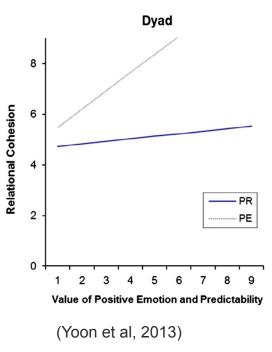
Positive emotion was more important for group cohesion in dyads

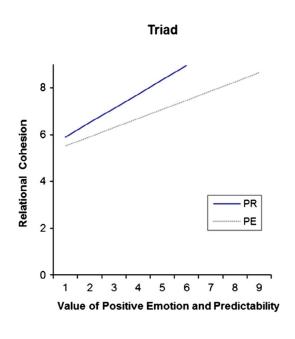
Predictability of exchange partners was more important for group cohesion in triads



In short, triads are the smallest social group.

Our aim is to understand the properties of a triad as a starting point for understanding the network properties of larger social groups.





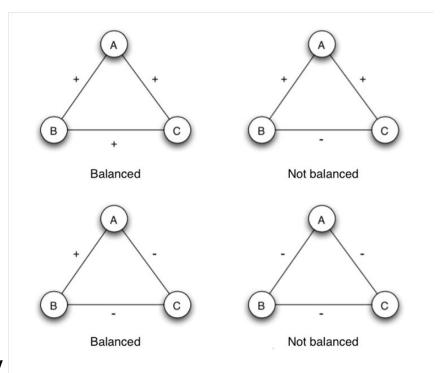
One of the earliest models of triads took inspiration from psychology

BALANCE THEORY



FRITZ HEIDER

- People try to avoid cognitive inconsistencies in their relationships
- This idea was applied to triads and developed to structural balance theory

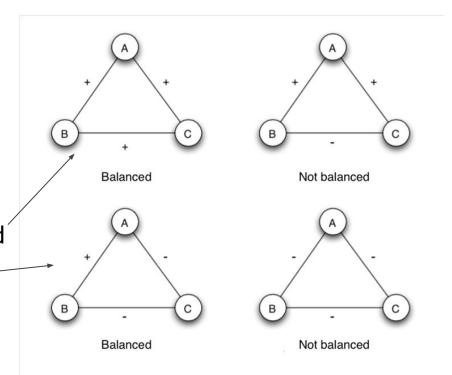


Friends: "+"

Enemies: "-"

Structural balance is achieved when

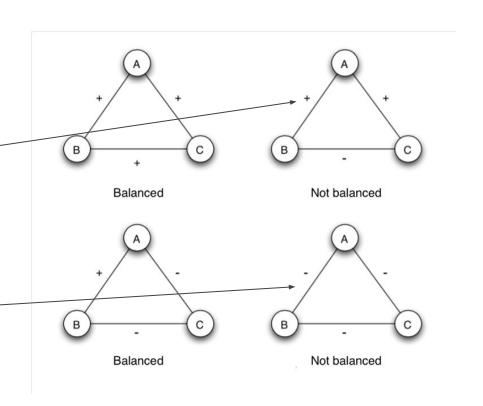
- A friend of my friend is also my friend
- An enemy of my friend is my enemy



Triad is unbalanced when it consists of an **odd number of negative edges**

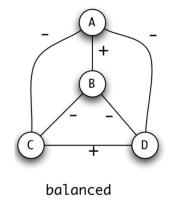
One negative edge: My two friends are enemies

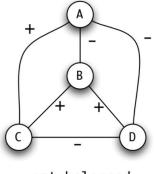
Three negative edges: The enemy of my enemy is also my enemy —



We can extend this logic of balance in triads to an entire graph

- →Are all triads in a graph structurally balanced?
- → Does every triad have an even number of negative ties?

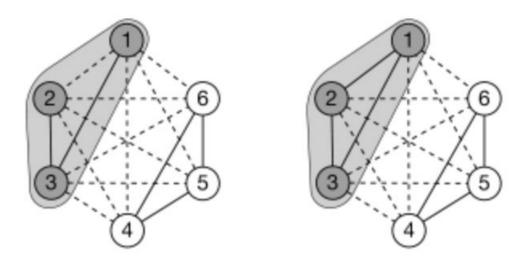




not balanced

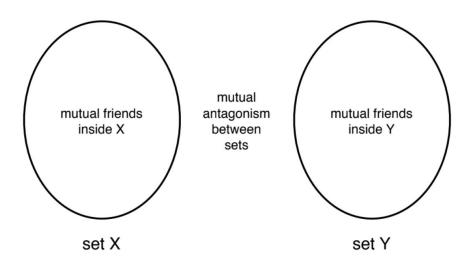
A graph with even a single unbalanced triad is "unbalanced"

For these completely connected graphs with positive (solid) and negative (dashed) edges, which one is balanced?

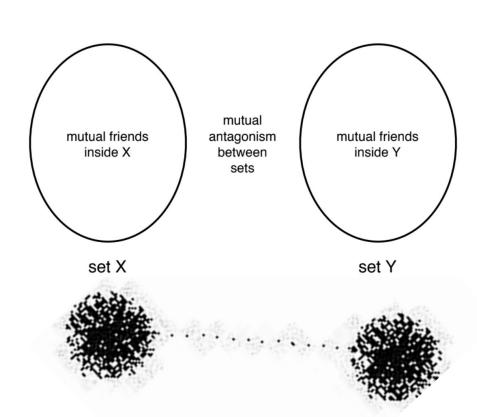


A completely connected graph is balanced iff:

- (a) all pairs of nodes are friends or
- (b) all nodes can be divided into two groups X and Y where everyone in the same group are friends and all the pairs between groups are enemies

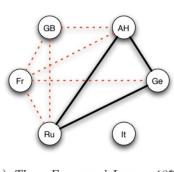


A completely connected, structurally balanced graph implies perfect polarization

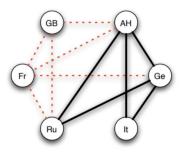


Toward structural balance in the real world:

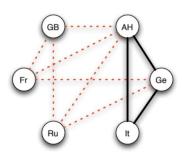
The evolution of alliances among nations in 19th century Europe



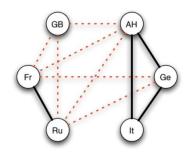
(a) Three Emperors' League 1872–81



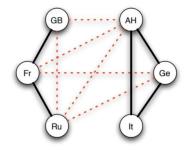
(b) Triple Alliance 1882



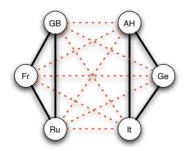
(c) German-Russian Lapse 1890



(d) French-Russian Alliance 1891–94



(e) Entente Cordiale 1904



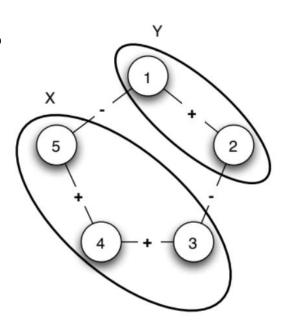
(f) British Russian Alliance 1907

What about graphs that are not completely connected?

Balance can be defined for signed graphs in general as the following:

Nodes can be divided into two groups X and Y where all the edges in X are positive, all edges in Y are positive, and all edges between X and Y are negative

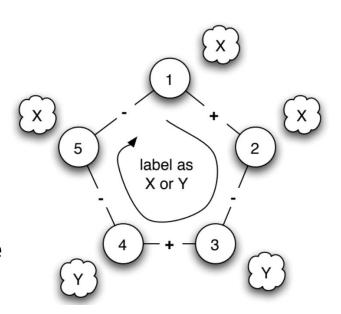
(Not all dyads need to be connected)



How to check for balance:

Search for a cycle of any length that has an odd number of negative edges \rightarrow Unbalanced

Start from a starting node and label the adjacent node according to the sign of the edge



Structural balance is an "ideal type," a model against which empirical social networks can be evaluated

Measures of structural balance

- Line index of balance: How far away is a given graph from perfect balance?
- The minimum number of edges whose signs need to be changed to obtain perfect balance

- Cycle index for balance: Proportion of positive cycles in a signed graph

Balance theory is a useful starting point for understanding factions in small groups, group structure, and polarization

As we will see in later lectures, many models of political polarization are based, at their core, on the logic of structural balance

The Reality is More Complex

Most real world social networks are not perfectly balanced

- Signed relationships are rare
 - → usually, presence/absence of an edge is more adequate

Triads in Directed Graphs

Directed Edges Create More Complexity

Four triadic isomorphism classes exist in an undirected triad



Many different triadic relationships exist in directed networks

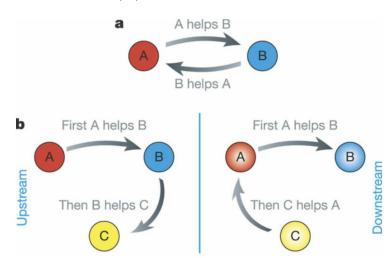
Example: structure of paying respect

- A→B→C: Hierarchical
- $A \rightarrow B \rightarrow C \rightarrow A$: Cyclical
- A→B←C: Competitive

An Example of Three-Way Relations: Reciprocity

Example: direct reciprocity (a) vs. generalized reciprocity (b)

- Generalized reciprocity is reciprocity in one direction
- The receiver does not give back to the giver, but pays it forward
- Represented as cycles (what goes around comes around)
- Such a configuration can require high levels of trust among group members



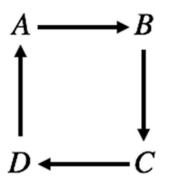


Generalized Reciprocity

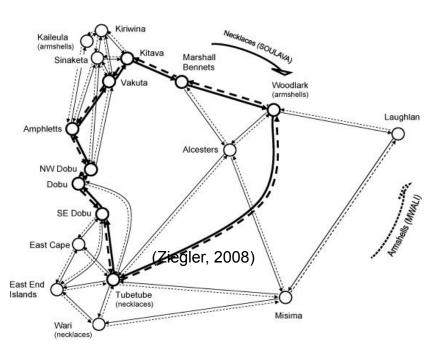
A more flexible triad representation is needed to study diverse network phenomena

Example:

Systems of generalized reciprocity: Kula ring

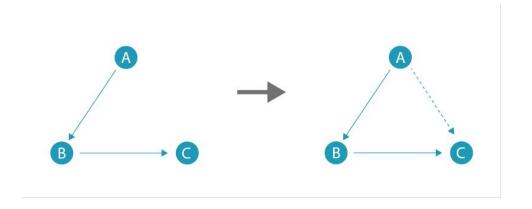






A prevalent triadic pattern in most social networks is transitivity

Triad of A, B, and C is transitive if whenever $\mathbf{A} \rightarrow \mathbf{B}$ and $\mathbf{B} \rightarrow \mathbf{C}$, then $\mathbf{A} \rightarrow \mathbf{C}$



Transitivity is the most basic structural representation of hierarchy and dominance orders (military, pecking order, monkey grooming)

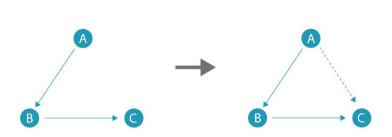


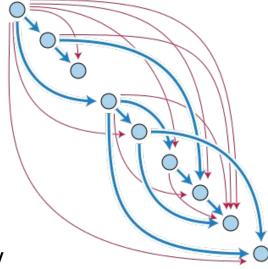






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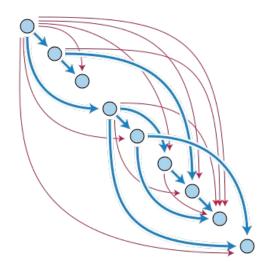




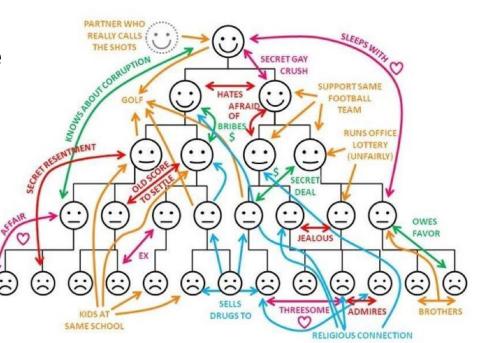
It is the building block of larger structures of hierarchy in networks

Transitivity is also an ideal type

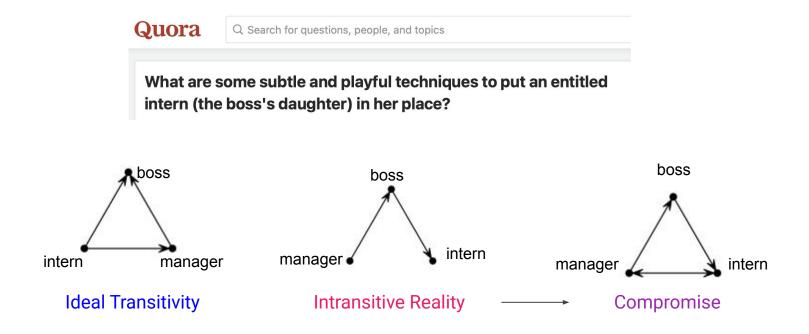
Perfectly transitive networks are rare



REAL ORGANIZATION CHART



We know when a situation deviates from perfect transitivity

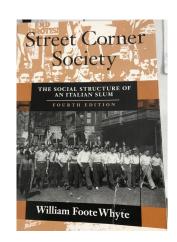


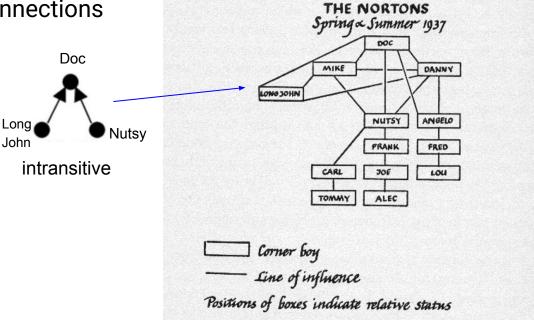
An individual's status tends to be fragile without transitivity

Long John is high in status because of his connections to other high status

members, but without the connections

to the lower ranks





Cycles and transitive triads are part of 16 triad **isomorphism classes**

Built on combinations of dyad isomorphism classes (M, A, N)

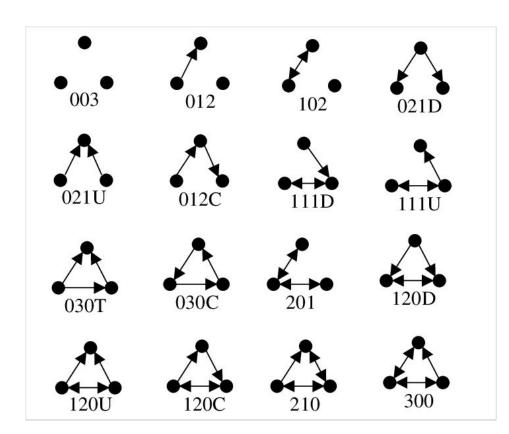
Class names are based on the number of M, A, N dyads

Triad census counts the frequency of these 16 isomorphism classes in a network

021U 030C 030T 120C

Mutual: 0, Asymmetric: 3, Null: 0

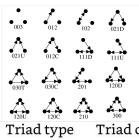
Just like the dyad census (M, A, N dyads), the triad census can give clues to the **social forces / mechanisms** that may be prevalent in a given social network



Just like the dyad census (M, A, N dyads), the triad census can give clues to the **social forces** / **mechanisms** that may be prevalent in a given social network

Example:

Networks where the 030T triads are prevalent are probably hierarchical in nature

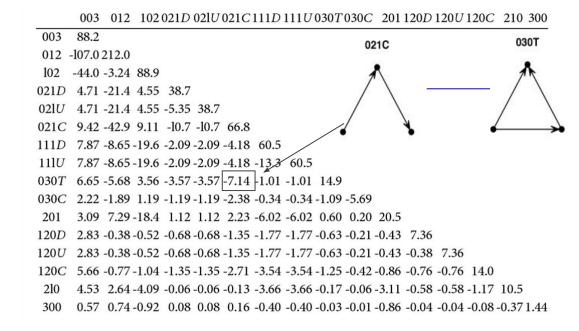


120U 120C 21	300	35-W 25-W 25-W 25-W	200-200 Pag 200-200-200 No PAGE
Triad type	Triad census	Expected value	Standard deviation
003	376	320.06	9.39
012	366	416.82	14.56
102	143	171.19	9.43
021D	114	44.09	6.22
021 <i>U</i>	34	44.09	6.22
021 <i>C</i>	35	88.17	8.17
111D	39	73.74	7.78
111U	101	73.74	7.78
→030 <i>T</i>	23	18.17	3.86
030 <i>C</i>	0	6.06	2.39
201	20	28.97	4.52
120 <i>D</i>	16	7.74	2.71
120U	25	7.74	2.71
120 <i>C</i>	9	15.48	3.74
210	23	12.38	3.25

We can also study what structural tendencies characterize a network by looking at the covariance / correlation between different isomorphism classes

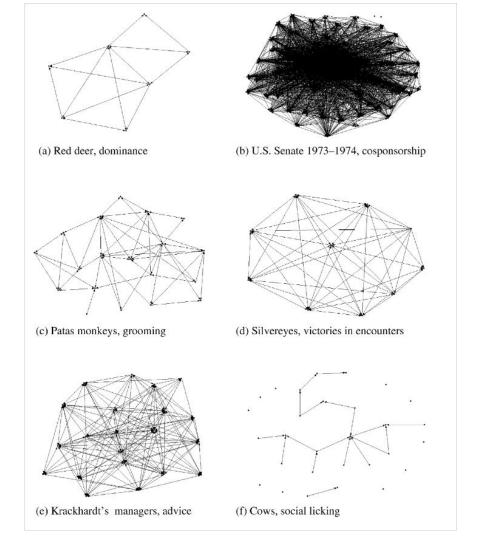
Which classes show high correlation and why?

Covariance matrix of a friendship network



One can gain insight about a community or social group by **comparing the distributions** of the 16 isomorphism classes across different networks

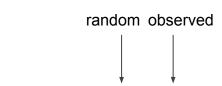
But, direct comparison of the observed isomorphism classes is problematic because networks have different size, density, and dyad distributions (M, A, N)



An apples-to-apples comparison involves comparing the empirical network to a random graph of the same size and same number of dyad isomorphism classes (M, A, N)

By quantifying the deviation from random, we can compare different networks through this deviation

 Interpretation: Relative to a random network, observed network deviates by X amount



Triad	Formula	34 Group		35 Group		44 Group		45 Group	
Type		Theor.	Real	Theor.	Real	Theor.	Real	Theor.	Real
		(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
003	$(1-\Delta)^6$	0.15	1.05	0.00	0.00	17.80	29.26	26.02	38.00
012	$6\Delta(1-\Delta)^5$	1.80	2.10	0.12	0.29	35.60	23.97	39.27	25.94
102	$3\Delta^2(1-\Delta)^4$	1.75	2.45	0.26	1.62	5.93	12.79	4.94	11.95
021D	$3\Delta^2(1-\Delta)^4$	1.75	9.09	0.26	0.74	5.93	6.47	4.94	5.94
021U	$3\Delta^2(1-\Delta)^4$	1.75	1.40	0.26	0.59	5.93	3.24	4.94	2.44
021C	$6\Delta^2(1-\Delta)^4$	3.49	0.00	0.52	0.15	11.87	1.91	9.88	1.06
111D	$6\Delta^3(1-\Delta)^3$	6.78	0.35	2.21	2.21	3.96	0.59	2.49	0.96
111U	$6\Delta^3(1-\Delta)^3$	6.78	12.59	2.21	4.26	3.96	11.91	2.49	7.04
030T	$6\Delta^3(1-\Delta)^3$	6.78	6.99	2.21	1.32	3.96	1.03	2.49	1.01
030C	$2\Delta^3(1-\Delta)^3$	2.26	0.00	0.74	0.15	1.32	0.00	0.83	0.00
201	$3\Delta^4(1-\Delta)^2$	6.58	2.80	4.69	5.88	0.66	4.26	0.31	2.44
120D	$3\Delta^4(1-\Delta)^2$	6.58	2.80	4.69	4.85	0.66	0.00	0.31	0.07
120U	$3\Delta^4(1-\Delta)^2$	6.58	23.08	4.69	8.09	0.66	2.06	0.31	1.50
120C	$6\Delta^4(1-\Delta)^2$	13.16	0.35	9.38	2.94	1.32	0.15	0.63	0.07
210	$6\Delta^5(1-\Delta)$	25.55	17.48	39.71	33.53	0.44	1.18	0.16	0.86
300	Δ^6	8.27	17.48	28.03	33.38	0.02	1.18	0.01	0.71

Triad Census

Limitations of triad census:

Triads are **not independent** of one another

- A change in a single arc affects the isomorphism classes of N-2 triplets

Quantification of the observed isomorphism classes is deterministic

- What if the observed triad distribution is one instantiation of a probabilistic tie generating mechanism?

More sophisticated **parametric statistical models** (e.g., Exponential Random Graph Models) are available

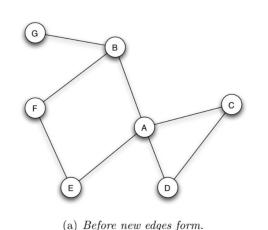
- Controls for size, density, dyad types, and other higher order subgraphs
- These models enable asking questions such as, "controlling for the number of cycles in this network, how much effect does transitivity have on tie formation?

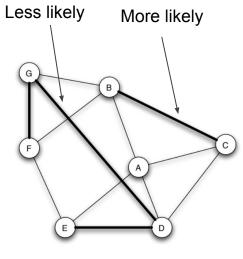
Triadic Closure and Clustering

A social network is constantly evolving

- New ties form
- Existing ties decay

In this process, two nodes that are connected to the same set of other nodes have a higher probability of forming an edge

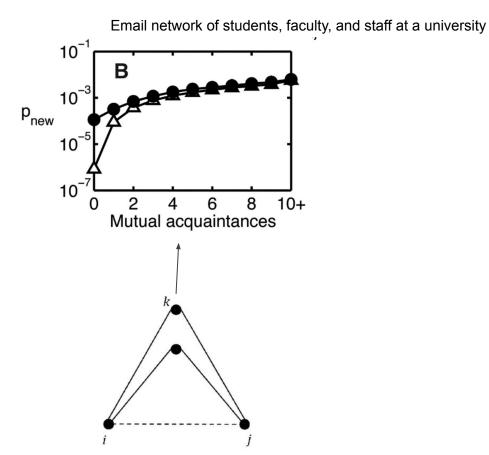




(b) After new edges form.

This results in a high frequency of fully connected triads in a network.

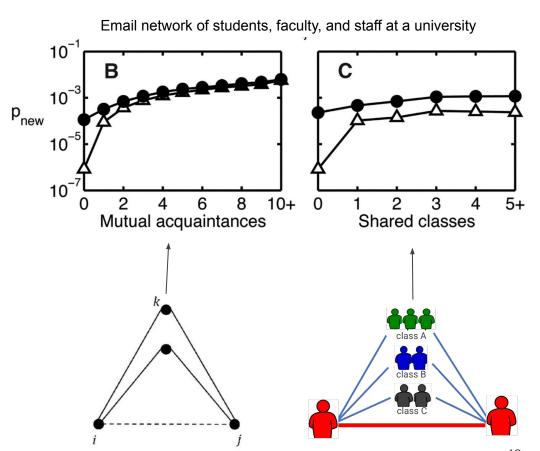
The more friends in common, the more likely a new tie forms



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The more friends in common, the more likely a new tie forms

This effect is stronger when the dyad also shares similar social contexts (e.g., two students overlap in multiple classes)



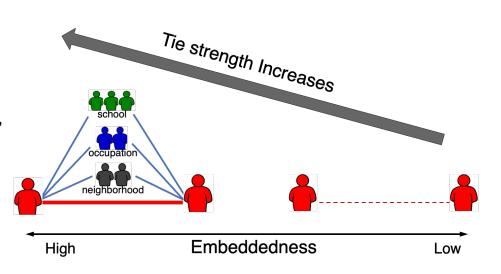
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Furthermore, an *i-j* tie that closes many triads:

- is less likely to dissolve
- tends to be stronger (higher interaction frequency, positive emotions, reciprocity)

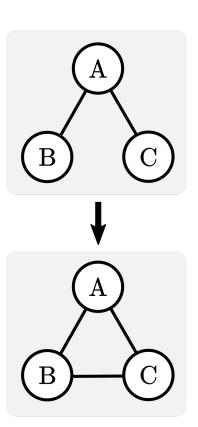
The *i-j* tie is highly **embedded** in common network neighbors

→ Recall the stabilizing force of triads that Simmel argued



Why do social networks exhibit strong triadic closure?

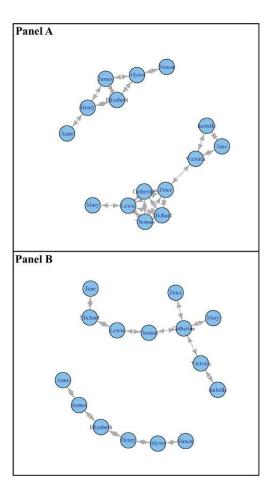
- **Opportunity**: If B and C each often spend time with A, they are likely to meet each other
- **Trust**: B and C can trust each other because they trust A
- Incentives: A introduces B to C because A believes more can be achieved if the three collaborate
- Cognitive dissonance: If A is good buddies with B and C, A
 may be distressed by the cognitive dissonance → Establish
 structural balance by introducing the two
- Homophily: All three share a common interest, so A is friends with B and C. There is a good chance B and C meet each other even without A's introduction (What a small world!)



Our brain may have evolved to perceive triadic closure

People more accurately remember who is connected to whom in a network with more triadic closure

→ Triadic closure is a sort of information compression heuristic, which may have given an evolutionary advantage



Case Study: Clustering Coefficient

Measurement of Triadic Closure

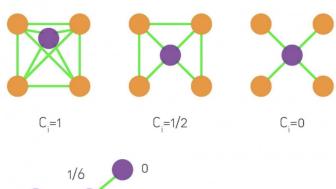
The extent of triadic closure in a network:

 Local clustering coefficient: The probability that two neighbors of a node are connected

Number of ties among *i*'s neighbors (excluding ties involving *i*)

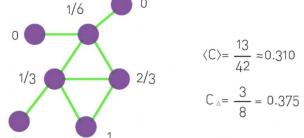
$$C_i = \frac{2L_i}{k_i(k_i - 1)}$$

The average across *all* nodes is that network's "local" clustering coefficient



a.

b.



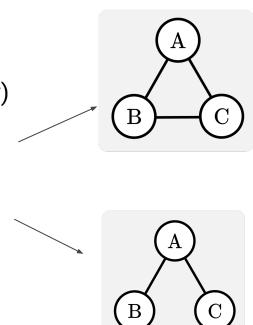
Measurement of Triadic Closure

The extent of triadic closure in a network:

Global clustering coefficient (i.e., transitivity)

$$C_{\Delta} = \frac{3 \times Number\ Of\ Triangles}{Number\ Of\ Connected\ Triples}$$

This is the ratio of the number of closed triads to the number of open triads in a network



What do you see?

Network	Nodes (N)	Links (<i>L</i>)	Average path length $(\langle \ell angle)$	Clustering coefficient (C)
Facebook Northwestern Univ.	10,567	488,337	2.7	0.24
IMDB movies and stars	563,443	921,160	12.1	0
IMDB co-stars	252,999	1,015,187	6.8	0.67
Twitter US politics	18,470	48,365	5.6	0.03
Enron email	87,273	321,918	3.6	0.12
Wikipedia math	15,220	194,103	3.9	0.31
Internet routers	190,914	607,610	7.0	0.16
US air transportation	546	2,781	3.2	0.49
World air transportation	3,179	18,617	4.0	0.49
Yeast protein interactions	1,870	2,277	6.8	0.07
C. elegans brain	297	2,345	4.0	0.29
Everglades ecological food web	69	916	2.2	0.55

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High clustering in many human social networks

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Huh???

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Bipartite network: links only between movies and stars

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IMDB movies and stars	Movies			0
IMDB co-stars	1.13 . 13.5			0.67
Twitter US politics				0.03
Enron email				0.12
Wikipedia math				0.31
Internet routers	Actors (0.16
US air transportation	546	2,781	3.2	0.49
World air transportation	3,179	18,617	4.0	0.49
Yeast protein interactions	1,870	2,277	6.8	0.07
C. elegans brain	297	2,345	4.0	0.29
Everglades ecological food web	69	916	2.2	0.55

Huh???

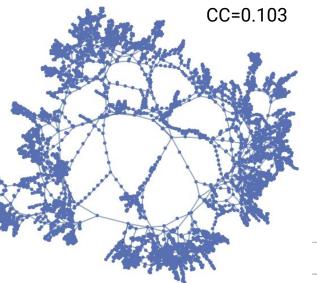
Network	Nodes (N)	Links (<i>L</i>)	Average path length $(\langle \ell angle)$	Clustering coefficient (C)
Facebook Northwestern Univ.	10,567	488,337	2.7	0.24
IMDB movies and stars	563,443	921,160	12.1	0
IMDB co-stars	252,999	1,015,187	6.8	0.67
Twitter US politics	18,470	48,365	5.6	0.03
Enron email	87,273	321,918	3.6	0.12
Wikipedia math	15,220	194,103	3.9	0.31
Internet routers	190,914	607,610	7.0	0.16
US air transportation	546	2,781	3.2	0.49
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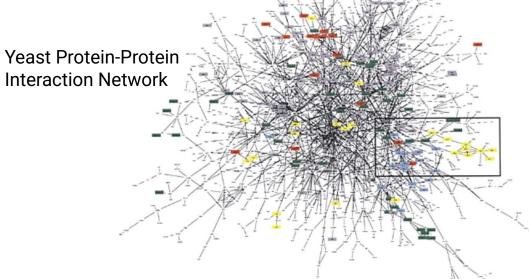
Retweet cascade trees look like stars (B rt A, C rt B \rightarrow C rt A)

Network	Nodes (N)	Links (<i>L</i>)	Average path length $(\langle\ell angle)$	Clustering coefficient (C)
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Yeast protein interactions	1,870			0.07
C. elegans brain	297			0.29
Everglades ecological food web	69	910	۷.۷	0.55

Many non-human networks don't cluster

U.S. Powergrid network





Measure	Т	AP	HM	IS-PCI	I Other data sets		
Measure	"Small"	"Medium"	"Small"	"Medium"	Y2H	DIP	TP
Nodes n	193(15)	1,365(1,250)	99(7)	1,544(1,501)	1,870	1,788	434
Interactions I	191(38)	3,230(3,150)	67(7)	3,481(3,456)	2,240	3,003	868
Connectance C	0.01(0.36)	0.003(0.004)	0.01(0.33)	0.003(0.003)	0.001	0.002	0.009
Clustering cc	0.248(0.66)	0.216(0.233)	0.071(0)	0.048(0.049)	0.068	0.188	0.054
Diameter <i>D</i>	(1.94)	(4.93)	(1.81)	(4.41)			
Longest path	(4)	(12)	(3)	(11)			
Stretch parameter b	0.78	0.48	0.65	0.34	0.34	0.53	0.55

Case Study: Signed Networks in Social Media

Reminder: We saw signed undirected networks

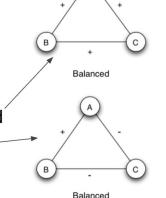
Structural Balance: Triads of Friends and Enemies

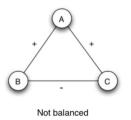
Friendships: "+"

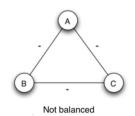
Enemy relations: "-"

Structural balance is achieved when

- A friend of my friend is also my friend
- An enemy of my friend is my enemy -







10

Reminder: And we saw unsigned directed networks

Transitivity

A prevalent triadic pattern in most social networks is transitivity

Triad of A, B, and C is transitive if whenever $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$



26

Competing theories

Structural balance theory:

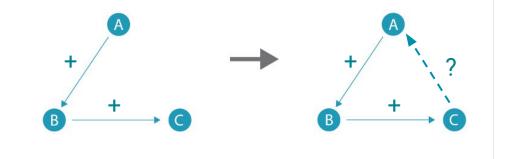
Triangles with three positive signs (three mutual friends) and those with one positive sign (two friends with a common enemy) are more plausible — and hence should be more prevalent in real networks — than triangles with two positive signs (two enemies with a common friend) or none (three mutual enemies).

Status theory:

 A positive directed link indicates that the creator of the link views the recipient as having higher status; and a negative directed link indicates that the recipient is viewed as having lower status.

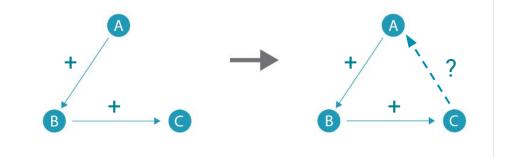
C→A sign?

- Balance theory:
- Status theory:



C→A sign?

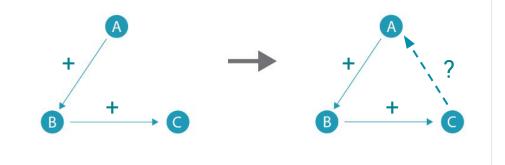
- Balance theory: +
- Status theory: -



C→A sign?

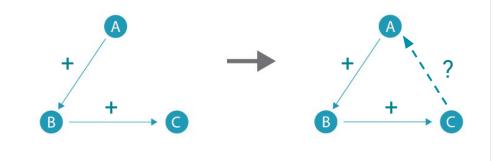
- Balance theory: +
- Status theory: -

Empirical approach?



C→A sign?

- Balance theory: +
- Status theory: -



Empirical approach:

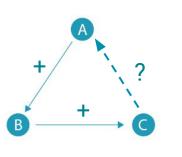
- Compute whether the observed number of triads of each type is overrepresented or underrepresented compared to chance.
- Contrast that with the predictions made by the balance and status theories.

Highlights

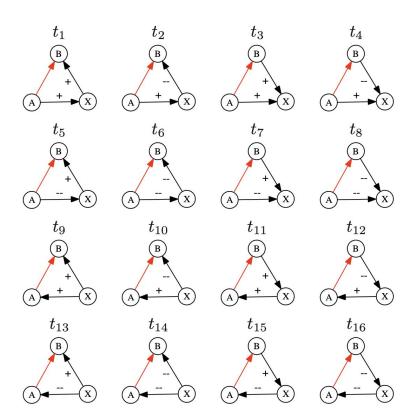
Three networks: online ratings site (trust / distrust), Slashdot discussions ("friends" and "foes"), and Wikipedia admin candidates (votes for / against).

Significant alignment between the observed network data and (weak) structural balance theory:

- triangles with exactly two + edges (two enemies with a common friend) are massively underrepresented in the data relative to chance.
- triangles with three + edges (three mutual friends) are massively overrepresented.



Highlights



1		$D(\perp)$	_	-	D	D	C	C
t_i	count	P(+)	s_g	s_r	B_g	B_r	S_g	S_r
t_1	178,051	0.97	95.9	197.8	\checkmark	\checkmark	\checkmark	\checkmark
t_2	45,797	0.54	-151.3	-229.9	✓	\checkmark	\checkmark	0
t_3	246,371	0.94	89.9	195.9	✓	\checkmark	0	\checkmark
t_4	25,384	0.89	1.8	44.9	0	0	\checkmark	\checkmark
t_5	45,925	0.30	18.1	-333.7	0	\checkmark	\checkmark	\checkmark
t_6	11,215	0.23	-15.5	-193.6	0	0	\checkmark	\checkmark
t_7	36,184	0.14	-53.1	-357.3	✓	\checkmark	\checkmark	\checkmark
t_8	61,519	0.63	124.1	-225.6	✓	0	\checkmark	\checkmark
t_9	338,238	0.82	207.0	-239.5	✓	0	\checkmark	\checkmark
t_{10}	27,089	0.20	-110.7	-449.6	√	\checkmark	\checkmark	\checkmark
t_{11}	35,093	0.53	-7.4	-260.1	0	0	\checkmark	\checkmark
t_{12}	20,933	0.71	17.2	-113.4	0	\checkmark	\checkmark	\checkmark
t_{13}	14,305	0.79	23.5	24.0	0	0	\checkmark	\checkmark
t_{14}	30,235	0.69	-12.8	-53.6	0	0	\checkmark	0
t_{15}	17,189	0.76	6.4	24.0	0	0	0	\checkmark
t_{16}	4,133	0.77	11.9	-2.6	✓	0	✓	0
	Nu	ımber of o	correct pre	dictions	8	7	14	13

(Leskovec, Huttenlocher, & Kleinberg, CHI 2010)

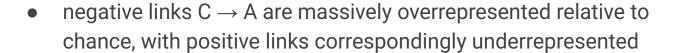
Highlights

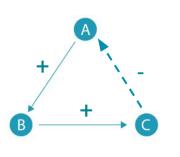
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- triangles with three + edges (three mutual friends) are massively overrepresented.

But, status theory is more effective at explaining local patterns of signed links:





Summary

A unique feature of social networks is the dynamics of triads

- Triad is the most elementary group
- Structural balance
- Transitive and cyclical triads
- Triad census
- Triadic closure
- Measures of clustering